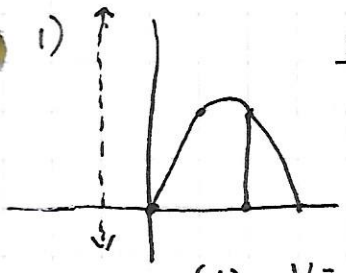
AP Calculus BC

Volume of Solids

1)



SHELLS

$V = \int 2\pi r h \, dx$

$r = x - (-1) = x + 1$

$h = 3x - x^2$

(A) $V = \int_0^3 2\pi (x+1)(3x-x^2)$

2) $Y_1 = \sqrt{x} \quad Y_2 = e^{-x}$

$Y_1 = Y_2$ @ $x = 0.426 \rightarrow B$

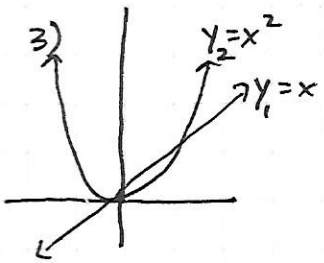
a) $\text{Area} = \int_0^B (Y_2 - Y_1)\,dx = 0.1615$

b) $V = \pi \int_0^B (Y_2+1)^2 dx - \pi \int_0^B (Y_1+1)^2 dx$

$= 1.6305$

c) $V = \frac{\pi}{8} \int_0^B (Y_2 - Y_1)^2 dx = 0.0347$

3)



a) $V = \pi \int_0^1 (x^2 - x^4)\,dx = 0.418$ or 0.419

b) $V = \pi \int_0^1 (x - (-1))^2 dx - \pi \int_0^1 (x^2 - (-1))^2 dx$

$= \pi \int_0^1 \left[(x+1)^2 - (x^2+1)^2 \right] dx = 1.466$

c) $V = \pi \int_0^1 (7 - x^2)^2 dx - \pi \int_0^1 (7-x)^2 dx$

$= \pi \int_0^1 \left[(7-x^2)^2 - (7-x)^2 \right] dx = 6.911$ or 6.912

SHELLS d) $V = \int_0^1 2\pi r h \, dx$

$V = 2\pi \int_0^1 x (x - x^2)\,dx = 0.523$ or 0.524

SHELLS e) $V = 2\pi \int_0^1 (x+1)(x-x^2)\,dx = 1.570796327$

SHELLS f) $V = 2\pi \int_0^1 (17-x)(x-x^2)\,dx = 17.27875959$